

Polycube Shape Space

Hui Zhao, Xuan Li, Wencheng Wang, Xiaoling Wang, Shaodong Wang, Na Lei, Xianfeng Gu

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*e-mail: whn@ios.ac.cn, *tel: 010-62661611

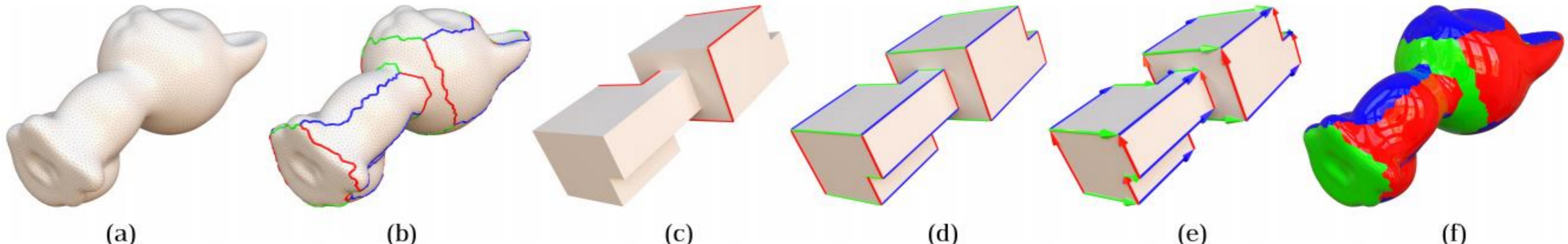
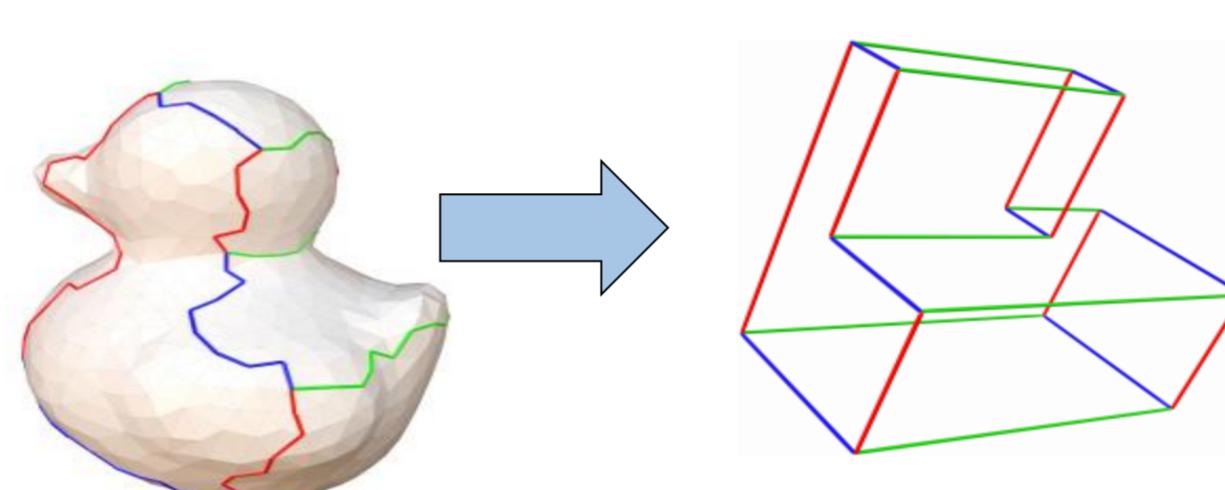


Figure 1: (a) the "kitten" mesh of genus one, (b) the polycube graph, (c) the homology loops, (d) the arc colorization, (e) the initial directions, (f) the polycube topology.

The problem:

1. Is there a polyhedron whose graph is the given one?
2. If so, how to generate the polyhedron?
3. Any applications in mesh processing for this kinds of relationship?



Our Contribution:

A linear system to generate a space of polycube polyhedra from a graph.

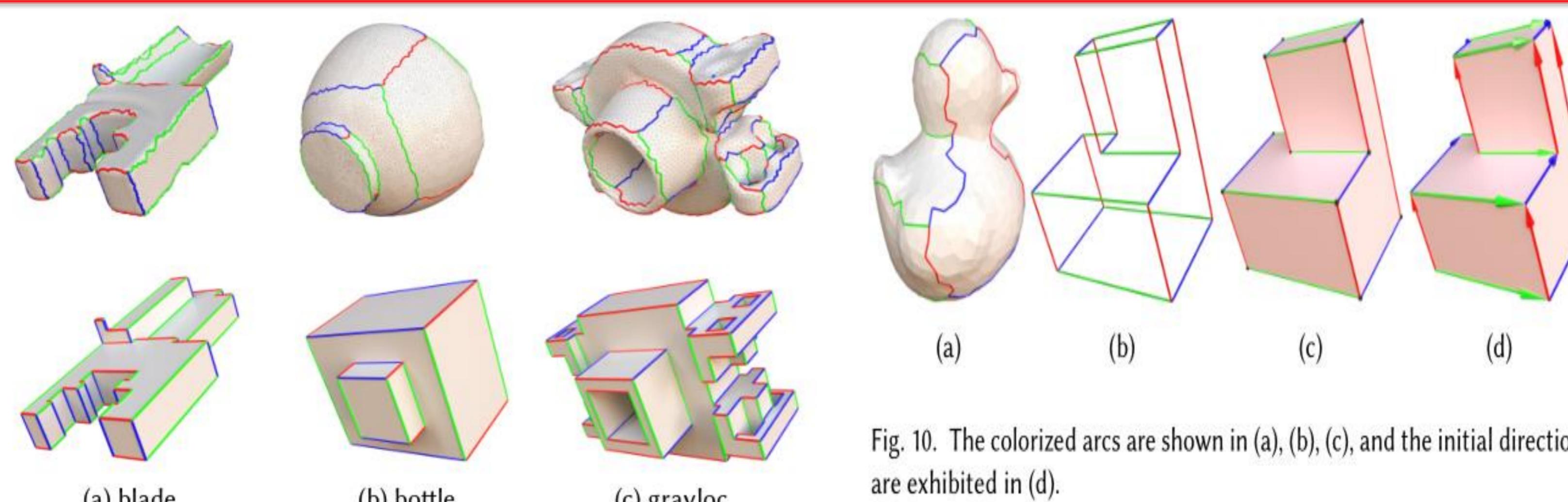


Fig. 10. The colorized arcs are shown in (a), (b), (c), and the initial directions are exhibited in (d).

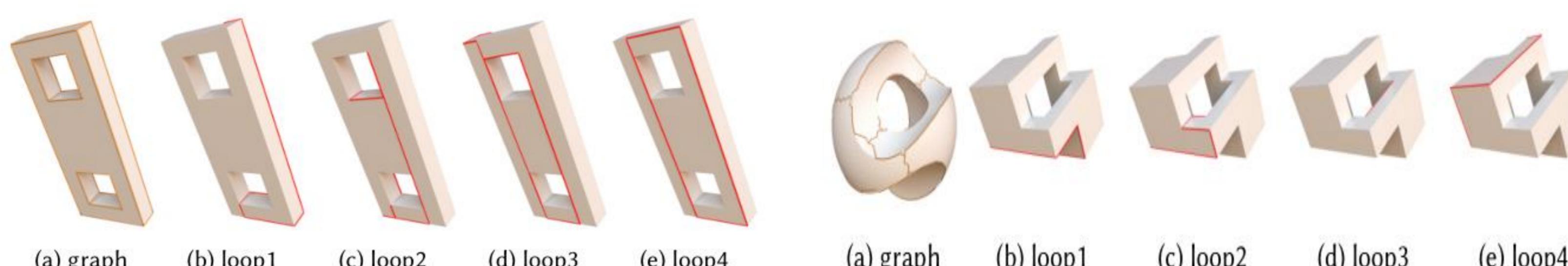


Fig. 12. The homology loops of the "double torus" model.

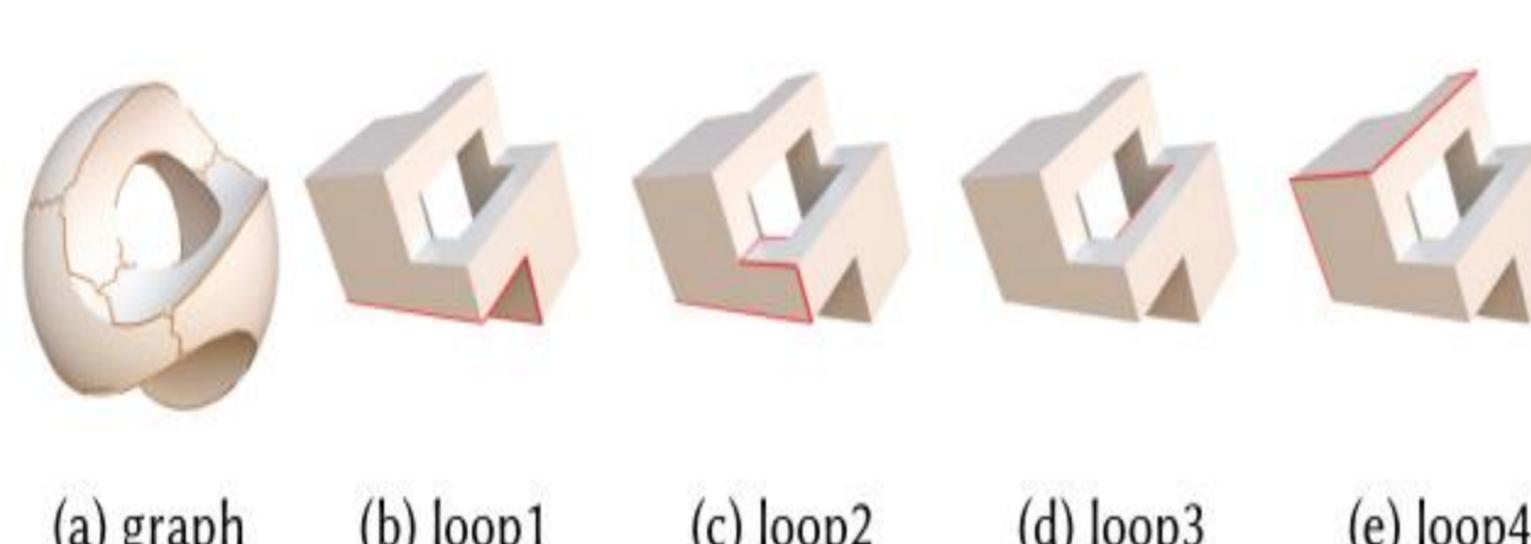


Fig. 13. The homology loops of the "sculpt" model.

Table 1. The dimension of polycube shape spaces

Model	V	E	F	V'	E'	F'	genus	dimension
airplane	52	78	28	52	78	28	0	25
armadillo	236	354	120	236	354	120	0	117
armchair	24	36	14	24	36	14	0	11
bimba	68	102	38	72	110	40	0	35
bone	40	60	22	40	60	22	0	19
bunny	84	126	44	84	126	44	0	41
coverrear	72	108	44	84	132	50	0	41
david	90	135	47	90	135	47	0	44
dente	44	66	24	44	66	24	0	21
dino2	106	159	55	106	159	55	0	52
hand	48	72	26	48	72	26	0	23
max	64	96	35	66	100	36	0	32
pear	24	36	16	28	44	18	0	13
pensatore	136	204	70	136	204	70	0	67
sphinx	72	108	38	72	108	38	0	35
bottle	52	78	29	58	90	32	1	26
camel	284	426	144	288	434	146	1	141
dragon	210	315	105	210	315	105	1	102
kitten	46	69	24	48	73	25	1	21
rocker	66	99	36	72	111	39	1	33
teaport	48	72	28	56	88	32	1	25
cup	32	48	18	40	64	22	2	15
dtorus	40	60	22	48	76	26	2	19
eight	24	36	14	32	52	18	2	11
sculpt	40	60	18	40	60	18	2	15
block	48	72	26	60	96	32	3	23
elephant	176	264	85	178	268	86	3	82
holes3	32	48	18	44	72	24	3	15
kiss	146	219	70	148	223	71	3	67
Deckel	80	120	40	92	144	46	4	37
fertility	104	156	46	104	156	46	4	43
sculpture	108	162	51	114	174	54	4	48
botijo	154	231	71	158	239	73	5	68
dancing	216	324	101	230	352	108	8	98

We proved:

Theorem 4.2 For a polycube graph with the number of nodes $|N|$, arcs $|A|$, patches $|P|$ respectively, the dimension of its polycube shape space is at least $|P| - 3$.

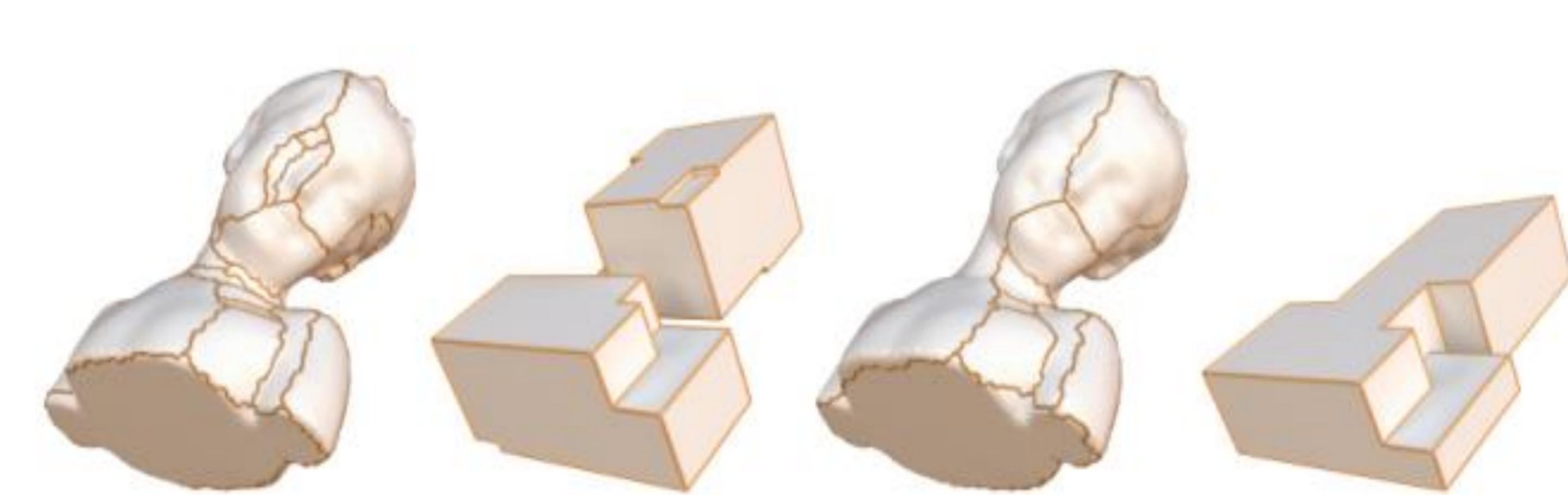
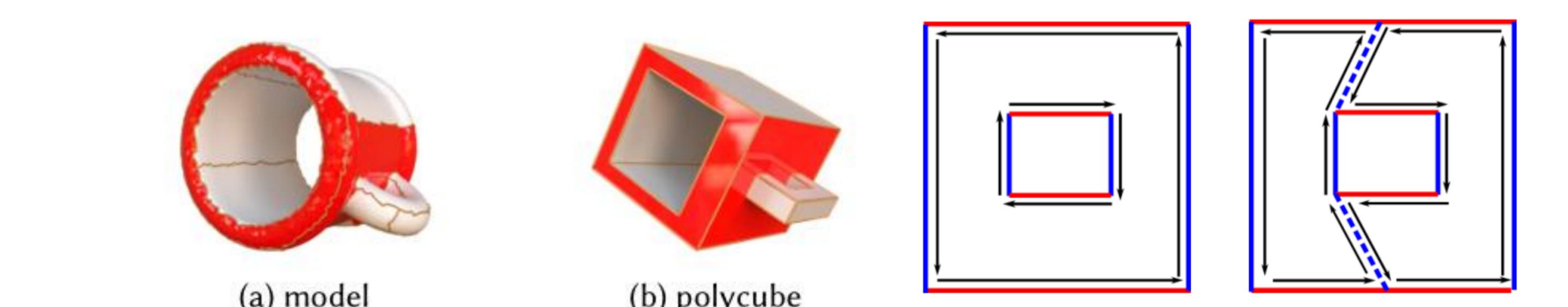
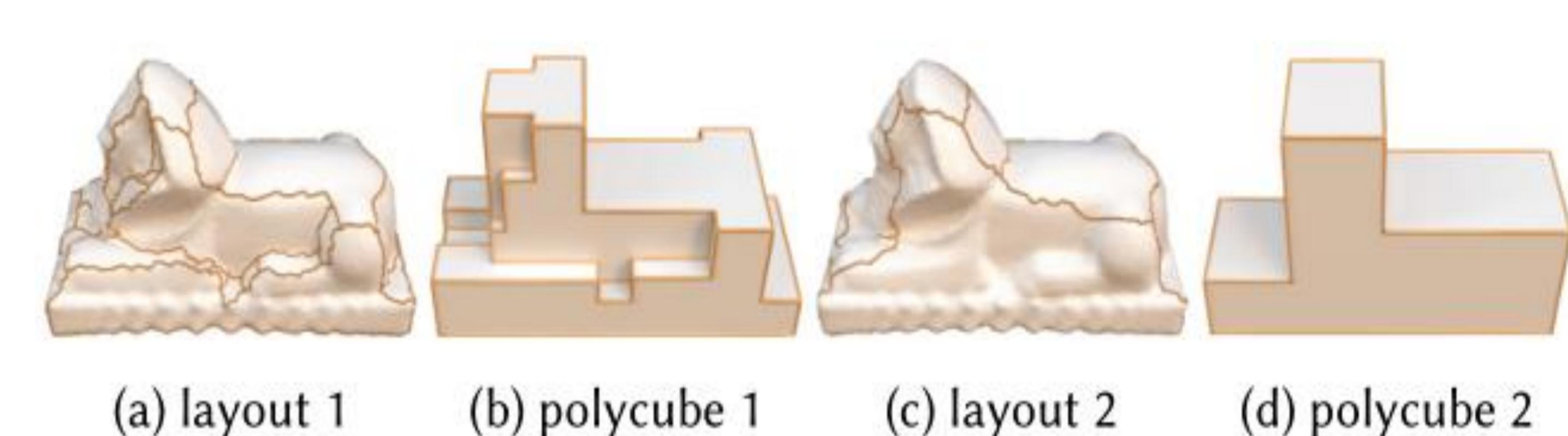


Figure 2: (a) the original mesh; (b) the polycube graph embedded on the mesh; all patches are colored according to their orientations (c), (d); the polycube polyhedron (e); the polycube mesh (f).



Polycube Applications: quad-meshing, hex-meshing

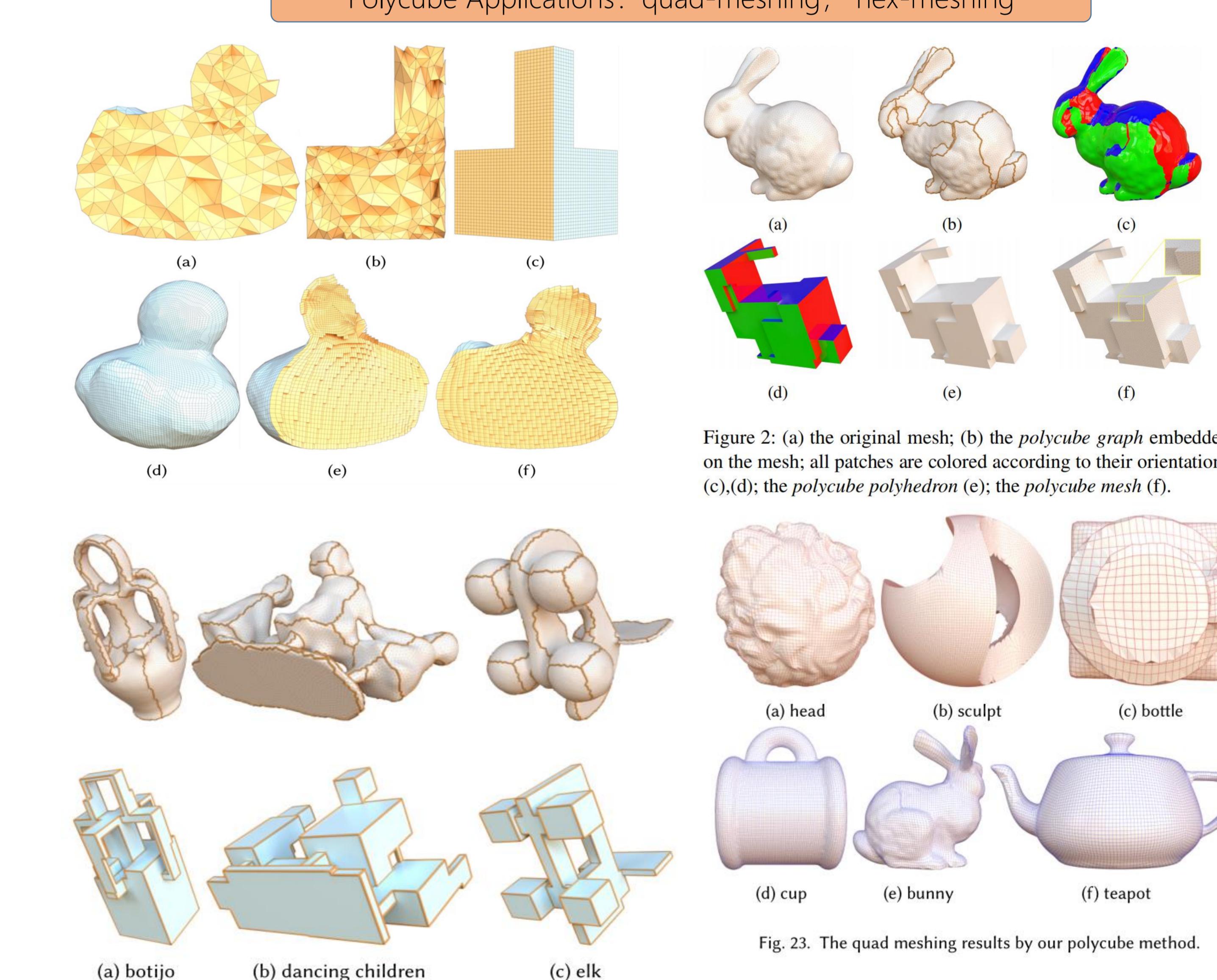


Fig. 23. The quad meshing results by our polycube method.

[Zie12] ZIEGLER, GÜNTER M. *Lectures on polytopes*. Vol. 152. Springer Science & Business Media, 2012 [\[1\]](#) [\[2\]](#)

[GKKZ67] GRÜNBAUM, BRANKO, KAIBEL, VOLKER, KLEE, VICTOR, and ZIEGLER, GÜNTER M. *Convex polytopes*. Vol. 1967. Springer, 1967 [\[1\]](#) [\[2\]](#)