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学术论文

Projective Peridynamic Modeling of Hyperelastic Membranes with Contact

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Introduction

Real-time simulation of hyperelastic membranes like cloth still faces a lot of challenges.

In this study, we propose projective peridynamics that uses a local-global strategy to enable fast and robust simulation of hyperelastic membranes with contact.

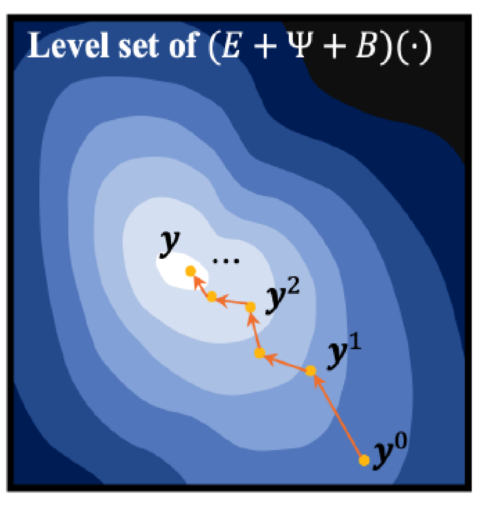
Our Contribution:

- A projective model based on peridynamics to simulate hyperelastic membranes with contact.
- A semi-implicit successive substitution method with co-dimensional extension to efficiently simulate hyperelastic materials in the global step.
- A gradient descent method to solve contact in the local step.

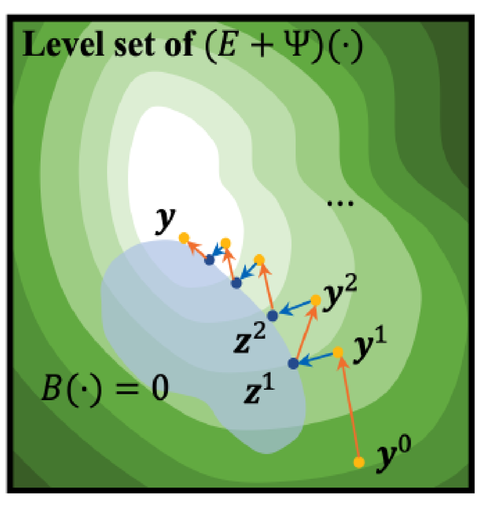
Methodology

Objective function: $\arg\min_{\mathbf{y}} \frac{1}{2h^2}\left\|\mathbf{M}^{\frac{1}{2}}(\mathbf{y}-\mathbf{y}^*)\right\|_F^2 + \Psi(\mathbf{y}) + \frac{1}{2}\|\mathbf{z}-\mathbf{y}\|_2^2 \quad s.t.\ B(\mathbf{z})=0$.

where $\mathbf{y}$ the position states, $\mathbf{M}$ the mass matrix, Ψ the hyper-energy and B the barrier energy of collision, h the timestep.



Traditional Newton Solver



Our L-G Solver

Framework

Algorithm 1: Projective Peridynamics

Input $\mathbf{y}^t$, $\mathbf{v}^t$, $\xi, h, \varepsilon, \hat{d}, s_0, k_b$
$\mathbf{y}^{k=0} \leftarrow \mathbf{y}^t + h\mathbf{v}^*$
$\mathbf{z}^{k=0} \leftarrow \mathbf{y}^t$
//We typically set $eps = 1e^{-4}$
while $\max_i \left(\left\|\mathbf{y}_i^k - \mathbf{y}_i^{k-1}\right\|_2\right) > eps$ *and* $k \leq max\_iter$ **do**
  foreach *vertex* i **do**
    Calculate $\mathbf{A}_i^k, \mathbf{s}_i^k, \mathbf{s}_i^t$ //Eq. (26)
    $\mathbf{y}_i^{k+1}$ = Jacobi($\mathbf{A}_i^k, \mathbf{s}_i^k, \mathbf{s}_i^t$) //Eq. (25)
  end
  Find active contact pairs.
  $\mathbf{z}^{k+1}$ = Project($\mathbf{y}^{k+1}, \mathbf{z}^k, \xi, h, \varepsilon, \hat{d}$) //Algorithm 2
  $\mathbf{y}^{k+1} \leftarrow \mathbf{z}^{k+1}$
end
$\mathbf{y}^{t+1} \leftarrow \mathbf{y}^k$

Global Step

Objective function:

$$\arg\min_{\mathbf{y}} \frac{1}{2h^2}\left\|\mathbf{M}^{\frac{1}{2}}(\mathbf{y}-\mathbf{y}^*)\right\|_F^2 + \Psi(\mathbf{y}),$$

which is equal to:

$$\mathbf{y}_i^{t+1} = \mathbf{y}_i^t + h\mathbf{v}_i^{t+1},$$

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^* + h\mathbf{M}_i^{-1}\mathbf{f}_i^{t+1},$$

$$\mathbf{f}_i = V_i \sum_j \left\{\underline{\mathbf{T}}_i\langle\mathbf{x}_j-\mathbf{x}_i\rangle - \underline{\mathbf{T}}_j\langle\mathbf{x}_i-\mathbf{x}_j\rangle\right\} V_j .$$

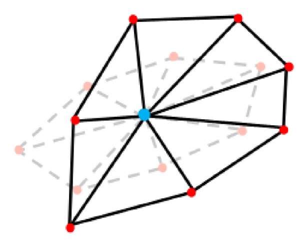
Semi-implicit separation:

$$\underline{\mathbf{T}}_i\langle\mathbf{x}_j-\mathbf{x}_i\rangle = \kappa_i\mathbf{U}_i\hat{\mathbf{P}}_i^+\left(\hat{\mathbf{F}}_i\right)\hat{\mathbf{F}}_i^{-1}\mathbf{U}_i^T\cdot(\mathbf{y}_j-\mathbf{y}_i) + \kappa_i\mathbf{U}_i\hat{\mathbf{P}}_i^-\left(\hat{\mathbf{F}}_i\right)\mathbf{V}_i^T\cdot(\mathbf{x}_j-\mathbf{x}_i),$$

Linearized equation:

$$\mathbf{y}_i^{k+1} = \left(m_i\mathbf{I}+\mathbf{A}_i^k\right)^{-1}\left(\sum_j \mathbf{A}_{ij}^k\mathbf{y}_j^k + \mathbf{s}_i^k + \mathbf{s}_i^t\right).$$

Stretching energy



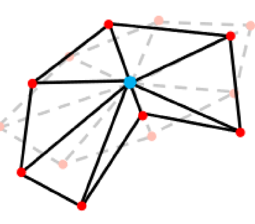
"membrane fiber" stretching

$$\Psi = s_0\left(\mathcal{A}_3(\lambda_1)+\mathcal{A}_3(\lambda_2)+\mathcal{A}_3(\lambda_3)\right)$$

$$\mathcal{A}_n = \frac{1}{n}\left(\frac{s^{n+1}-1}{n+1}+\frac{s^{-n+1}-1}{n-1}\right) \quad \text{(Xu, 2018)}$$

Remains to zero when isometric bending deformation exists . (λ is remains to 1.)

bending energy



"out-of-plane" bending

$$E_i^b = \frac{k_b}{2}\sum_j \|\mathbf{G}_i(\mathbf{x}_j-\mathbf{x}_i)\|_2^2$$

$$\mathbf{G} = \mathbf{F}^T\mathbf{F}^{-1}-\mathbf{I}$$

Remains to zero when only stretching deformation exists . ($\boldsymbol{F}$ is diagonal.)

Local Projection

Objective function: $\arg\min_{\mathbf{z}} \frac{1}{2}\left\|\mathbf{z}-\mathbf{y}^k\right\|_2^2, \quad s.t.\ B(\mathbf{z})=0.$

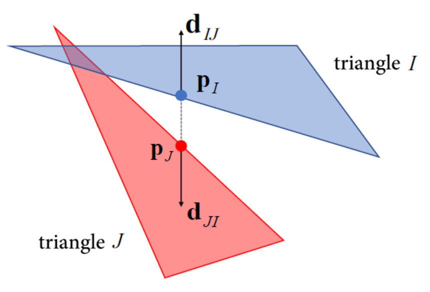
We solving the below one using gradient descend method:

$$\arg\min_{\mathbf{z}} \mathcal{B}(\mathbf{z}) := \left(\sum_i \frac{1}{2}\left\|\mathbf{z}_i-\mathbf{y}_i^k\right\|_2^2 + \mu\sum_c B_c(\mathbf{z})\right),$$

$$\lambda \leq \frac{\varepsilon\xi}{\mu\alpha_0\lceil|B'(d)|\rceil+\hat{d}},$$

$$\mathbf{g}_i = -\nabla_{\mathbf{z}_i}\mathcal{B} = \mu\sum_I \alpha_I^i\left[\frac{(d_{IJ}-\hat{d})^2}{d_{IJ}} + 2(d_{IJ}-\hat{d})\log\left(\frac{d_{IJ}}{\hat{d}}\right)\right], \quad \text{norm}(\mathbf{d}_{IJ}) + \mathbf{y}_i^k - \mathbf{z}_i$$

$$\mathbf{z}_i^{m+1} \leftarrow \mathbf{z}_i^m + \lambda\,\mathbf{g}_i.$$



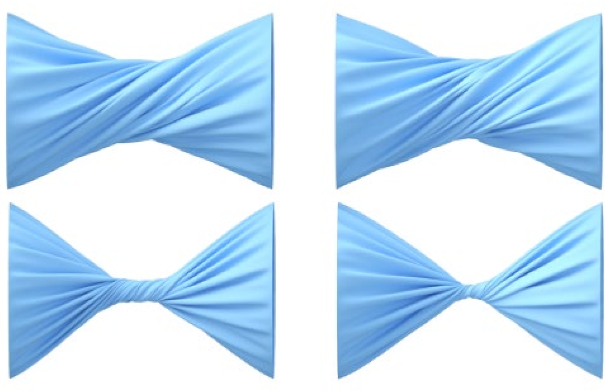
Gradient direction is exact the sum of proximal distance vectors.

Results

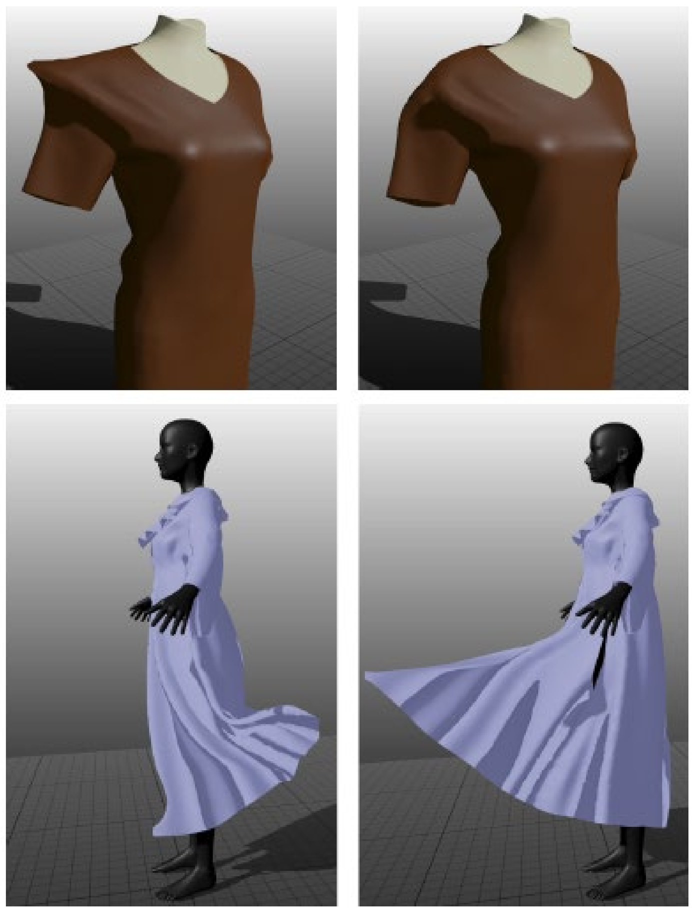
Bending stiffness

(a) 0× (b) 0.1× (c) 1× (d) 10×

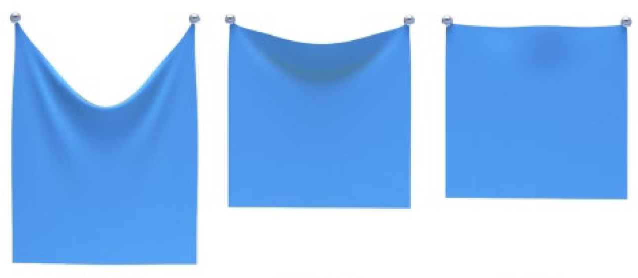
Thickness



Interactive-time clothes



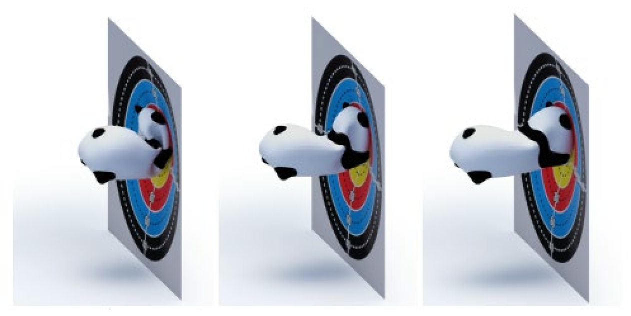
Stretching stiffness



(a) 0.1× (b) 1× (c) 10×

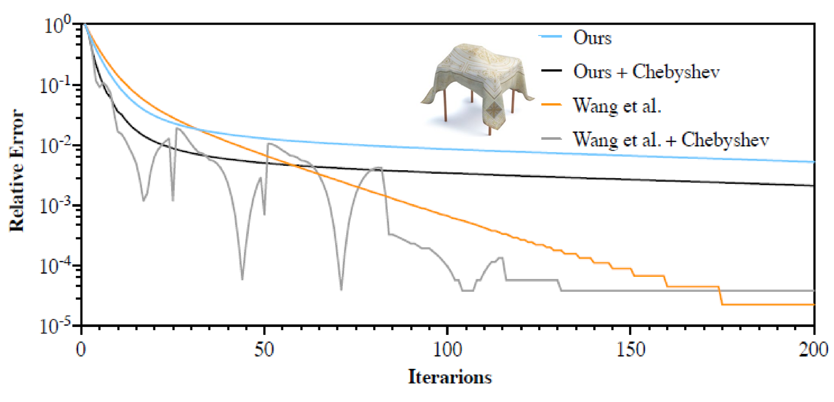
Multi-Layers

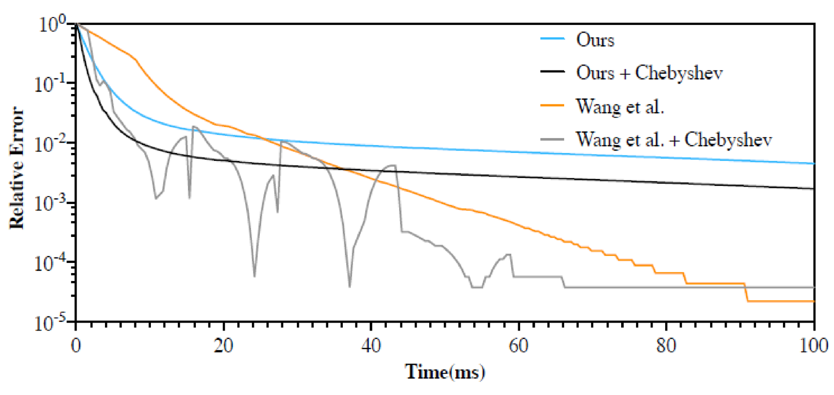
Contact and Separation



Comparison

Our method has better performance in the first sets of iterations. Our method also takes lower computational cost in taking one iteration due to the efficient analytical step length adjustment method, instead of back-tracking line search to seek descend step length which is used in Wang and Yang's [16] method.





Conclusion & Limitation

We present a stable and efficient semi-implicit successive substitution method for simulating hyperelastic membranes with contact based on peridynamics.

Limitations:

- the convergence rate slows down in simulating a multi-layer cloth;
- cannot implement strict controllable strain limits for elasticity when simulating shell-like materials.

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